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GRAVITATIONAL COLLAPSE AND BLACK HOLES ON THE BRANE

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Abstract

Gravitational collapse is assumed to be one of the main problems in general relativity and astrophysics. Using simple the Taylor expansion into the bulk and black string concept one can acquire acceptable "tidal charge" black hole and continue to explore "total tidal charge", physical mass and energy for the modified Friedmann model. Some numerical works of the interesting equations are implemented.

Keywords: Gravitational collapse, tidal charge, black hole, Friedmann model.

Introduction

The physics of brane-world compact objects and gravitational collapse is complicated by a number of factors, especially the confinement of matter to the brane, while the gravitational field can access the extra dimension, and the nonlocal (from the brane viewpoint) gravitational interaction between the brane and the bulk. Extra-dimensional effects mean that the 4D matching conditions on the brane, i.e., continuity of the induced metric and extrinsic curvature across the 2-surface boundary, are much more complicated to implement. High-energy corrections increase the effective density and pressure of stellar and collapsing matter. In particular this means that the effective pressure does not in general vanish at the boundary 2-surface, changing the nature of the 4D matching conditions on the brane. The nonlocal KK (Kaluza and Klein) effects further complicate the matching problem on the brane, since they in general contribute to the effective radial pressure at the boundary 2-surface. Gravitational collapse inevitably produces energies high enough, i.e., $\rho \gg \lambda$, to make these corrections significant. We expect that extra-dimensional effects will be negligible outside the high-energy, shortrange regime.

The contribution of the massive KK modes sums to a correction of the 4D potential. For $r \ll \ell$, one obtains

$$V\,(r) \approx \frac{GM\ell}{r^2}, \qquad (1)$$

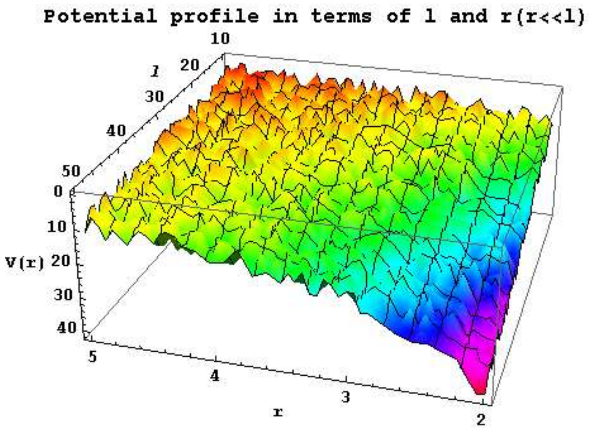

Figure 1 The Potential profile in terms of l and r ($r \ll \ell$)

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which simply reflects the fact that the potential becomes truly 5D on small scales.

For $r \gg \ell$,

$$V(r) \approx \frac{GM}{r}\left(1 + \frac{2\ell^2}{3r^2}\right), \tag{2}$$

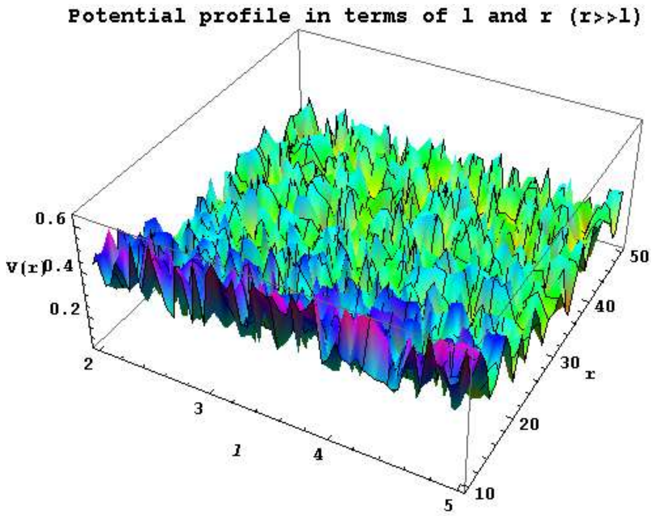

Figure 2 The Potential profile in terms of l and r ($r \gg \ell$)

which gives the small correction to 4D gravity at low energies from extra-dimensional effects. These effects serve to slightly strengthen the gravitational field, as expected.

A vacuum on the brane, outside a star or black hole, satisfies the brane field equations

$$R_{\mu\nu} = -\mathcal{E}_{\mu\nu}, \qquad R^{\mu}_{\mu} = 0 = \mathcal{E}^{\mu}_{\mu}, \qquad \nabla^{\nu}\mathcal{E}_{\mu\nu} = 0. \tag{3}$$

The Weyl term $\mathcal{E}_{\mu\nu}$ will carry an imprint of high-energy effects that source KK modes. This means that high-energy stars and the process of gravitational collapse will in general lead to deviations from the 4D general relativity problem. The weak-field limit for a static spherical source, Equation (2), shows that $\mathcal{E}_{\mu\nu}$ must be on zero, since this is the term responsible for the corrections to the Newtonian potential.

The black string

The projected Weyl term vanishes in the simplest candidate for a black hole solution. This is obtained by assuming the exact Schwarzschild form for the induced brane metric and "stacking" it into the extra dimension,

$$^{(5)}ds^2 = e^{-2|y|/\ell}\tilde{g}_{\mu\nu}dx^{\mu}dx^{\nu} + dy^2, \tag{4}$$

$$\tilde{g}_{\mu\nu} = e^{2|y|/\ell}g_{\mu\nu} = -(1 - 2GM/r)dt^2 + \frac{dr^2}{1 - 2GM/r} + r^2 d\Omega^2. \tag{5}$$

Each $\{y = \text{const.}\}$ surface is a 4D Schwarzschild spacetime, and there is a line singularity along $r = 0$ for all y. This solution is known as the Schwarzschild black string, which is clearly not localized on the brane $y = 0$. Although $^{(5)}C_{ABCD} \neq 0$, the projection of the bulk Weyl tensor along the brane is zero, since there is no correction to the 4D gravitational potential:

$$V\,(r) = \frac{GM}{r} \;\; \Rightarrow \; \mathcal{E}_{\mu\nu} \; = \; 0. \tag{6}$$

The violation of the perturbative corrections to the potential signals some kind of non-AdS$_5$ pathology in the bulk. Indeed, the 5D curvature is unbounded at the Cauchy horizon, as $y \to \infty$:

$$^{(5)}R_{ABCD}\,^{(5)}R^{ABCD} \; = \frac{40}{l^4} + \frac{48G^2M^2}{r^6} \; e^{4|y|/\ell}. \tag{7}$$

Taylor expansion into the bulk

One can use a Taylor expansion equation, in order to probe properties of a static black hole on the brane. For a vacuum brane metric,

$$\tilde{g}_{\mu\nu}(x,y) = \tilde{g}_{\mu\nu}(x,0) - \; \mathcal{E}_{\mu\nu}(x,0+)y^2 \; - \frac{2}{\ell}\mathcal{E}_{\mu\nu}(x,0+)|y|^3$$

$$+\frac{1}{12}\left[\mathcal{E}_{\mu\nu} \; - \frac{32}{\ell^2}\mathcal{E}_{\mu\nu} \; + \; 2R_{\mu\alpha\nu\beta}\mathcal{E}^{\alpha\beta} \; + \; 6\mathcal{E}_{\mu}{}^{\alpha}\mathcal{E}_{\alpha\nu}\right]_{y=0+} y^4 + \ldots \tag{8}$$

this shows in particular that the propagating effect of 5D gravity arises only at the fourth order of the expansion. For a static spherical metric on the brane,

$$\tilde{g}_{\mu\nu}dx^{\mu}dx^{\nu} \; = \; -F(r)dt^2 \; + \frac{dr^2}{H(r)} + \; r^2 d\Omega^2, \tag{9}$$

the projected Weyl term on the brane is given by

$$\mathcal{E}_{00} = \frac{F}{r}\left[H' \; - \frac{1-H}{r}\right], \tag{10}$$

$$\mathcal{E}_{rr} \; = \; -\frac{1}{rH}\left[\frac{F'}{F} - \frac{1-H}{r}\right], \tag{11}$$

$$\mathcal{E}_{\theta\theta} \; = \; -1 \; + \; H \; + \frac{r}{2}H(\frac{F'}{F} + \frac{H'}{H}). \tag{12}$$

These components allow one to evaluate the metric coefficients in Equation (8). For example, the area of the 5D horizon is determined by $\tilde{g}_{\theta\theta}$; defining $\psi(r)$ as the deviation from a Schwarzschild form for H, i.e.,

$$H(r) \; = \; 1 \; - \frac{2m}{r} + \; \psi(r), \tag{13}$$

where m is constant, one find

$$\tilde{g}_{\theta\theta}\,(r,y) = \; r^2 - \; \psi'^{\left(1+\frac{2}{\ell}|y|\right)y^2} + \frac{1}{6r^2}\left[\psi' + \frac{1}{2}(1 \; + \; \psi')(r\psi' \; - \; \psi)'\right] y^4 + \ldots \tag{14}$$

This shows how ψ and its r-derivatives determine the change in area of the horizon along the extra dimension. For the black string $\psi = 0$, and one has $\tilde{g}_{\theta\theta}(r,y) \; = \; r^2$. For a large black hole, with horizon scale $\gg \ell$, from Equation (2) that

$$\psi \; \approx \; -\frac{4m\ell^2}{3r^3}. \tag{15}$$

This implies that $\tilde{g}_{\theta\theta}$ is decreasing as we move off the brane, consistent with a pancake-like shape of the horizon. However, note that the horizon shape is tubular in Gaussian normal coordinates.

The "tidal charge" black hole

The equations (3) form a system of constraints on the brane in the stationary case, including the static spherical case, for which

$$\Theta = 0 = \omega_\mu = \sigma_{\mu\nu}, \qquad \dot{\rho}\mathcal{E} = 0 = q^{\mathcal{E}}_{\mu} = \dot{\pi}^{\mathcal{E}}_{\mu\nu}. \tag{16}$$

The nonlocal conservation equations $\nabla^\nu \mathcal{E}_{\mu\nu} = 0$ reduce to

$$\frac{1}{3}\vec{\nabla}_\mu \rho_{\mathcal{E}} + \frac{4}{3}\rho_{\mathcal{E}} A_\mu + \vec{\nabla}^\nu \pi^{\mathcal{E}}_{\mu\nu} + A^\nu \pi^{\mathcal{E}}_{\mu\nu} = 0, \tag{17}$$

where, by symmetry,

$$\pi^{\mathcal{E}}_{\mu\nu} = \Pi_{\mathcal{E}}\left(\frac{1}{3}h_{\mu\nu} - r_\mu r_\nu\right), \tag{18}$$

for some $\Pi_{\mathcal{E}}(r)$, with r_μ being the unit radial vector. The solution of the brane field equations requires the input of $\mathcal{E}_{\mu\nu}$ from the 5D solution. In the absence of a 5D solution, one can make an assumption about $\mathcal{E}_{\mu\nu}$ or $g_{\mu\nu}$ to close the 4D equations.

If one assume a metric on the brane of Schwarzschild-like form, i.e., $H = F$ in Equation (9), then the general solution of the brane field equations is

$$F = 1 - \frac{2GM}{r} + \frac{2G\ell Q}{r^2}, \tag{19}$$

$$\mathcal{E}_{\mu\nu} = -\frac{2G\ell Q}{r^4}\left[u_\mu u_\nu - 2r_\mu r_\nu + h_{\mu\nu}\right], \tag{20}$$

where Q is a constant. It follows that the KK energy density and anisotropic stress scalar are given by

$$\rho_{\mathcal{E}} = \frac{lQ}{4\pi\, r^4} = \frac{1}{2}\Pi_{\mathcal{E}}. \tag{21}$$

The solution (19) has the form of the general relativity Reissner–Nordstrom solution, but there is no electric field on the brane. Instead, the nonlocal Coulomb effects imprinted by the bulk Weyl tensor have induced a "tidal" charge parameter Q, where $Q = (M)$, since M is the source of the bulk Weyl field. We can think of the gravitational field of M being "reflected back" on the brane by the negative bulk cosmological constant. If one impose the small-scale perturbative limit ($r \ll \ell$) in Equation (1), one find that

$$Q = -2M. \tag{22}$$

Negative Q is in accord with the intuitive idea that the tidal charge strengthens the gravitational field, since it arises from the source mass M on the brane. By contrast, in the Reissner–Nordstrom solution of general relativity, $Q \propto +q^2$, where q is the electric charge and this weakens the gravitational field. Negative tidal charge also preserves the spacelike nature of the singularity, and it means that there is only one horizon on the brane, outside the Schwarzschild horizon:

$$r_h = GM\left[1 + \sqrt{1 - \frac{2\ell Q}{GM^2}}\right] = GM\left[1 + \sqrt{1 + \frac{4\ell}{GM}}\right]. \tag{23}$$

The tidal-charge black hole metric does not satisfy the far-field r^{-3} correction to the gravitational potential, as in Equation (2), and therefore cannot describe the end-state of collapse.

However, Equation (19) shows the correct 5D behavior of the potential ($\propto r^{-2}$) at short distances, so that the tidal-charge metric could be a good approximation in the strong-field regime for small black holes.

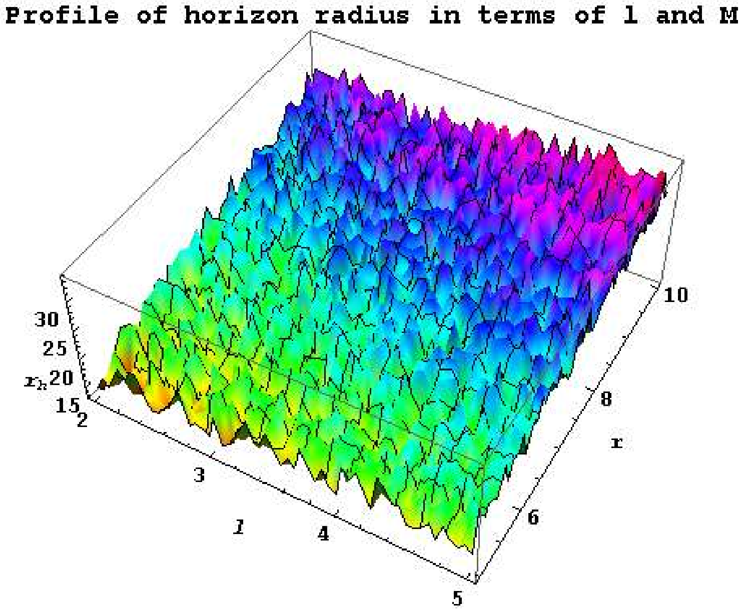

Figure 3 The profile of horizon radius in terms of l and M

It has not considered any back-reaction on the brane metric, and the same flux will reasonably be seen by a distant observer for whom ∂τ asymptotically becomes a time-like Killing vector. However, the surplus energy must be released, since no BW or GR model can explain the Weyl anomaly, and this directly implies that probably holds only for a short time about the formation of the horizon.

Gravitational collapse on the brane

In this section we will study a continuous model for the gravitational collapse. One consider a Tolman-like model with a central (Oppenheimer-Snyder model) OS core. The star is therefore described as a cloud of dust with falling off continuous density and no sharp boundary. The classical four-dimensional behavior will be recovered in the limit of negligible star density (with respect to the brane vacuum energy density λ).

The BW effective four-dimensional Einstein equations with vanishing cosmological constant on the brane as

$$G_{\mu\nu} = 8\pi T^{eff}_{\mu\nu} \tag{24}$$

Here one have

$$T^{eff}_{\mu\nu} = \rho^{eff} u_\mu u_\nu + p^{eff} h_{\mu\nu} + q^{eff}_{(\mu u_\nu)} + \pi^{eff}_{\mu\nu}, \tag{25}$$

where u^μ is the unit four-velocity of matter ($u^\mu u_\mu = -1$), $h_{\mu\nu}$ the space-like metric that projects orthogonally to u^μ ($h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$) and $\pi^{eff}_{\mu\nu}$ an anisotropic tensor.

For an isotropic perfect fluid, BW corrections to GR are described by the effective quantities

$$\rho^{eff} = \rho\,(1 + \frac{\rho}{2\lambda} + \frac{u}{\rho}) \tag{26}$$

$$P^{eff} = \rho\,(\frac{\rho}{2\lambda}(2p + \rho) + \frac{u}{3}) \tag{27}$$

$$q^{eff}_{\mu} = Q_{\mu} \tag{28}$$

$$\pi^{eff}_{\mu\nu} = \Pi_{\mu\nu,} \tag{29}$$

where ρ and p are the ("bare") energy density and pressure of matter. One also employed the following decomposition of the projection of the Weyl tensor on the brane (T.Clifon et al., 2017)

$$-\frac{1}{8\pi}\varepsilon_{\mu\nu} = u\left(u_{\mu}u_{\nu} + \frac{1}{3}h_{\mu\nu}\right) + Q_{\mu}u_{\nu} + Q_{\nu}u_{\mu} + \Pi_{\mu\nu}, \tag{30}$$

corresponding to an effective "dark" radiation on the brane with energy density U, pressure U/3, momentum density Q_{μ} and anisotropic stress $\Pi_{\mu\nu}$. Note that non-local bulk effects can contribute to effective imperfect fluid terms even when brane matter is a perfect fluid. Bianchi identities supplied by the junction conditions produce two kinds of conservation equations:

1. Local conservation equations (LCE):

$$\dot{\rho} + \theta(\rho + p) = 0 \tag{31}$$

$$D_a p + (\rho + p)A_a = 0 \tag{32}$$

2. Non-local conservation equations (NLCE's):

$$\dot{u} + \frac{4}{3}\theta u + D^a Q_a + 2A^a Q_a + \sigma^{ab}\Pi_{ab} = 0 \tag{33}$$

$$\dot{Q}_a + \frac{4}{3}\theta Q_a + \frac{1}{3}D_a u + \frac{4}{3}uA_a + D^b\Pi_{ab} + A^b\Pi_{ab} + \sigma^b_a Q_b - \omega^b_a Q_b = -\frac{\rho + p}{\lambda}D_a\rho \tag{34}$$

where D_a is the spatially projected derivative (defined by $D_a S^b{}_{......c} = h^e{}_a h^b{}_f \,..\,..h^g{}_c \nabla_e S^f{}_{...\,...g}$ for a = 1, 2, 3), $\theta = \nabla^{\alpha}u_{\alpha}$ the volume expansion, $\dot{S}^a{}_{......b} = u^{\alpha}\nabla_{\alpha}S^{a...}{}_{...b}$ the proper time derivative, $A_a = \dot{u}_a$ the acceleration, $\sigma_{ab} = D_{(a}u_{b)} - (\theta/3)\,h_{ab}$ the (traceless) shear, and $\omega_{ab} = -D_{[a}u_{b]}$ the vorticity.

Spherically symmetric dust

For the case with zero pressure (p = 0), that is dust on the brane, the quantities in Eqs. (26), (27) and (29) reduce to

$$\rho^{eff} = \rho\left(1 + \frac{\rho}{2\lambda}\right) + u \tag{35}$$

$$\rho^{eff} = \frac{\rho^2}{2\lambda} + \frac{u}{3} \tag{36}$$

$$\pi^{eff}_{\mu\nu} = \Pi_{\mu\nu} \tag{37}$$

Provided the matter density ρ does not vanish in the region of interest, one can use comoving coordinates in which $u^{\alpha} = (-1, 0, 0, 0)$. In the following, only consider the class of five-dimensional metrics which are diagonal (sufficiently close to the brane at y = 0) and spherically symmetric on the brane. In Gaussian normal coordinates, one can always write a bulk metric which is spherically symmetric on the brane as

$$ds^2 = -N^2(\tau, r, y)d\tau^2 + A^2(\tau, r\, y)dr^2 + 2B(\tau, r\, y)dt\, dr + R^2(\tau, r, y)d\Omega^2 + dy^2 \quad (38)$$

Upon using the restricted freedom to change the four- dimensional coordinates on the brane, one can always set $B(\tau, r, 0^+) = 0$ [3], so that the brane metric reads

$$ds^2{}_{y=0}\Big|^{+} = -N^2(\tau, r, 0^+)dr^2 + A^2(\tau, r, 0^+)dr^2 + R^2(\tau, r, 0^+)d\Omega^2 \quad (39)$$

Since just consider dust as brane matter, from the junction conditions at the brane, one also obtain

$$0 = K^{+}_{\tau,r}(\tau, r) \equiv \frac{1}{2}\frac{\partial g_{\tau r}}{\partial y}\Big|_{y=0^+} = \frac{\partial B}{\partial y}\Big|_{y=0^+} = 0. \quad (40)$$

Using the above result together with the bulk symmetry Z_2 with respect to the brane, we have $B(\tau, r, y) = y^2\,[V(\tau, r) + O(y)]$. Since the Weyl energy flux is related to B by

$$Q_a \sim \frac{\partial^2 B}{\partial y^2}\Big|_{y=0^+} \quad (41)$$

one finds that Q_a vanishes if $V(\tau, r) = 0$, which is in fact what we are assuming. The coefficient $g_{\tau r}$ then vanishes fast enough on the brane so that, from the five-dimensional Einstein equations

$$G_{AB} = -\Lambda g_{AB}, \quad (42)$$

in the limit $y \rightarrow 0^+$, one obtains the condition

$$0 = G_{\tau r}\big|_{y=0^+} = \frac{2}{NA}\left(\frac{\dot{A}}{A}\frac{R'}{R} + \frac{\dot{R}}{R}\frac{N'}{N} - \frac{\dot{R}'}{R}\right)_{y=0^+} \quad (43)$$

where a prime denotes ∂_r and a dot $\partial\tau$. Since our matter is pressure less, one can work in the proper time gauge $N(\tau, r, 0^+) = 1$ [3] and, using the residual gauge freedom in defining the radial coordinate r, one obtain

$$A(\tau, r, 0^+) = R'(\tau, r, 0^+). \quad (44)$$

This relation implies a Tolman geometry on the brane

$$ds^2 = -dr^2 + R'^2 dr^2 + R^2 d\Omega^2, \quad (45)$$

where $R = R(\tau, r)$ is a (generally non-separable) function of τ and r such that $4\pi R^2(\tau, r)$ equals he surface area of the shell comoving with dust particles located at the coordinate position r at the proper time τ. With the above symmetries, the vorticity, the acceleration and the Weyl energy flux vanish, $\omega_a = A_a = Q_a = 0$, and one obtain the simplified

LCE

$$\partial_\tau \rho + \theta\rho = 0 \quad (46)$$

And NLCE's

$$\partial_\tau u + \frac{4}{3}\theta u + \sigma^{ab}\Pi_{ab} = 0 \quad (47)$$

$$\frac{1}{3}D_a u + D^b\Pi_{ab} = -\frac{\rho}{\lambda}D_a\rho. \quad (48)$$

The volume expansion is also easily computed as

$$\theta = \partial_r \lfloor ln(R^2 \partial_r R) \rfloor = \frac{\partial_\tau \partial_r (R^3)}{\partial_r (R^3)}, \tag{49}$$

and for the shear one finds

$$\sigma_{ab} = \frac{1}{2} \partial_\tau h_{ab} - \frac{\theta}{3} h_{ab} , \tag{50}$$

where $h_{ab} = g_{ab}$ is the spatial part of the metric.

Conclusions

It has been attempted to show some of the key features of brane-world gravity from the perspective of astrophysics and cosmology, emphasizing a geometric approach to dynamics and perturbations. Inspired by the conjecture that classical black holes in the BW may reproduce the semi classical behavior of four-dimensional black holes, one has studied the gravitational collapse of a spherical star of dust in the RS scenario in order to clarify the underlying dynamics that leads to this interpretation. Regularity of the bulk geometry requires continuity of the matter stress tensor on the brane and can lead to a loss of mass from the boundary of the star. One has in particular shown that, excluding energy fluxes coming from the bulk Weyl tensor, a collapsing spherical star must have a spatially anisotropic, although isotropic in the angular directions, atmosphere, in order to have asymptotically flat solutions. Interestingly, such a feature is also present in the stress tensor of quantum fields on the Schwarzschild background. In visualizing of the results, fluctuation factor of 30 to 40 percent has been taken into consideration.

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